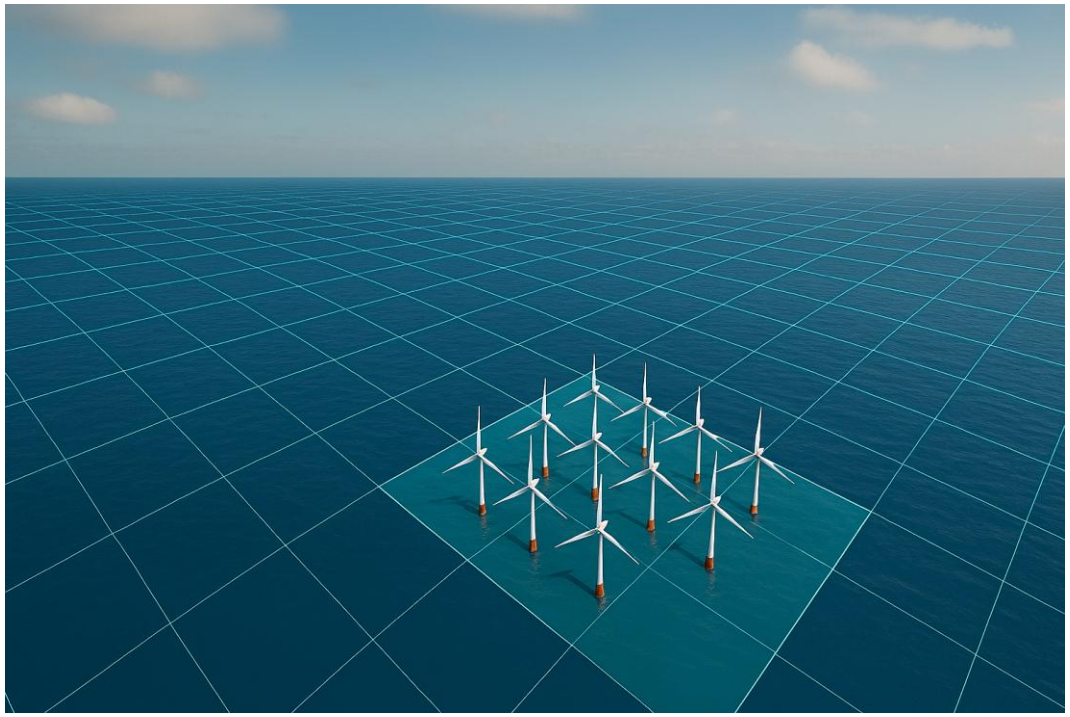
A Two Scale Momentum Wind Farm Parametrization

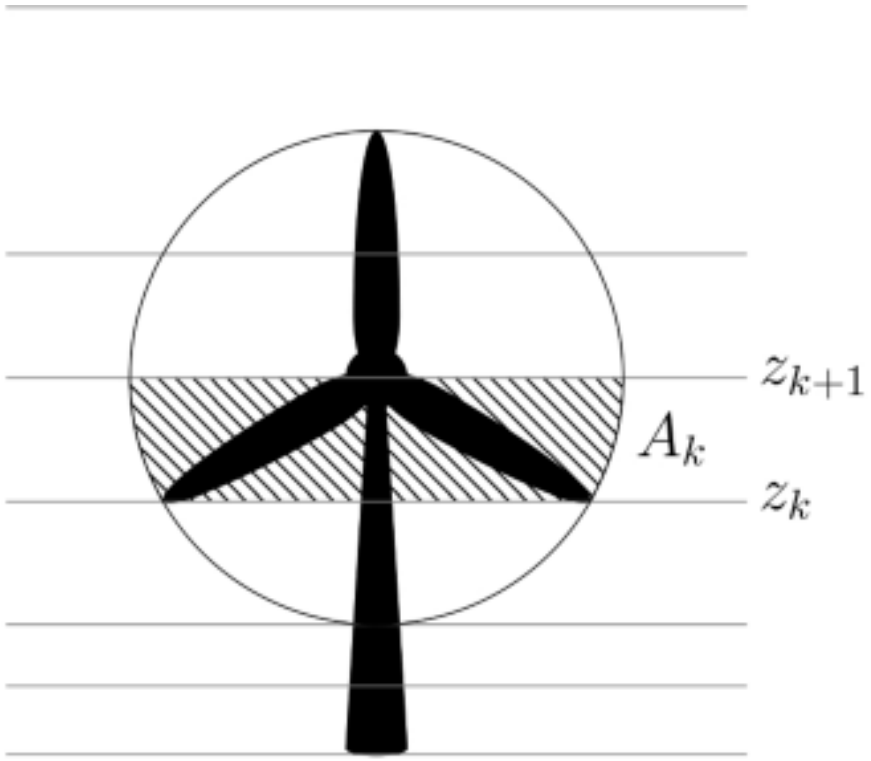
Oliver Ehlbeck, Koen Devesse, Ruben Borgers, Erik Delarue, Jan Helsen, Johan Meyers, Nicole Van Lipzig

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CLM Assembly 2026
23/09/2026

KU LEUVEN





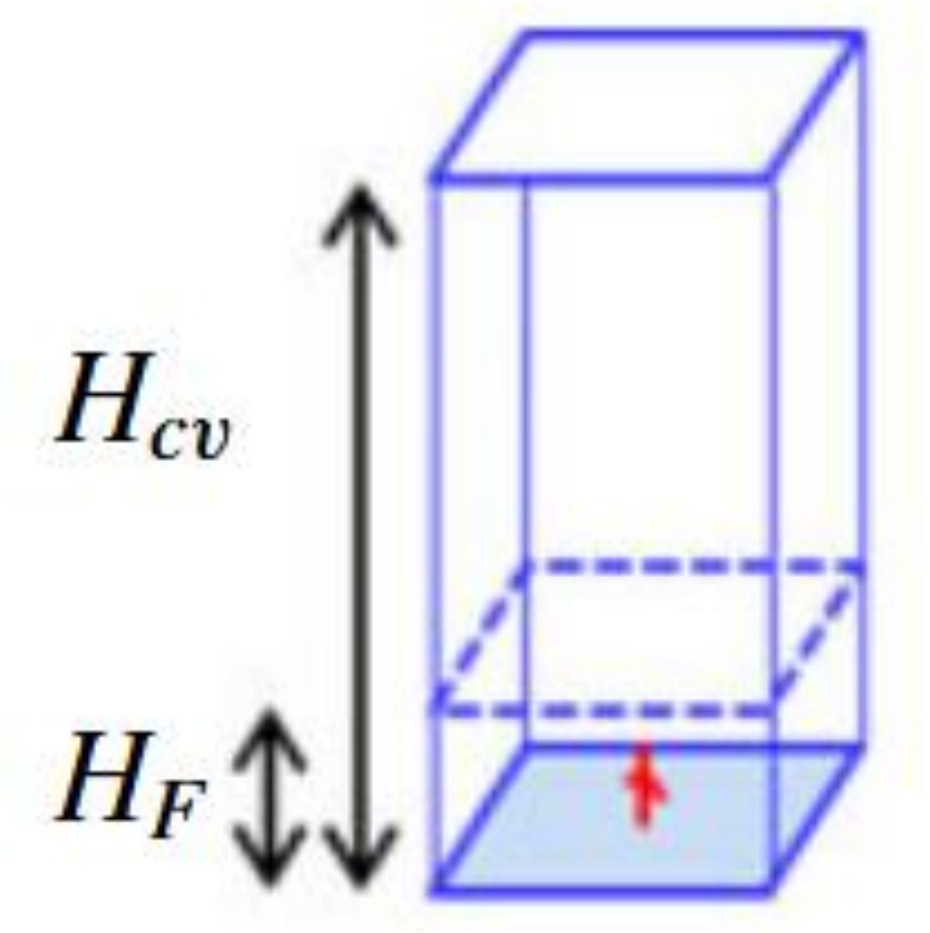
Fitch Parametrization

$$\partial_t \mathsf{TKE} = \mathsf{vertical\ transport} + \mathsf{shear\ force} + \mathsf{buyoancy} + \mathsf{dissiapation} + \partial_t \mathsf{TKE}_{\mathsf{drag}}$$

A two-scale Momentum Parametrization:

- Same core Idea as Fitch: source/sink terms
- Use a wake-accounting thrust coefficient
- Separation of scales: cell scale forces vs. turbine scale forces
- Different Philosophy:

thrust force for whole column (not layer by layer) and split it up



Equations of parametrization

Fitch:

$$\partial_t \mathrm{P}^{ij} = \frac{1}{2} N^{ij} \rho C_P \|V_{ij}^{hh}\|^3 A_F$$

$$\partial_t \mathrm{TKE}_{\mathrm{drag}}^{ijk} = \frac{1}{2} \frac{N^{ij} C_{TKE} \|V_{ijk}\|^3 A_{ijk}}{\Delta z_k} * ec^{ij}$$

$$C_p^* = \sum_{k=1}^{N_t} C'_{p,k} \frac{S_k^3}{U_f^3} \frac{A_k}{A_f}$$

$$U_{ij}^{REWS} := \sum_{k=1}^{k_b} (V_{ijk} \cdot \vec{e}_\phi) \frac{A_k}{A_F}$$

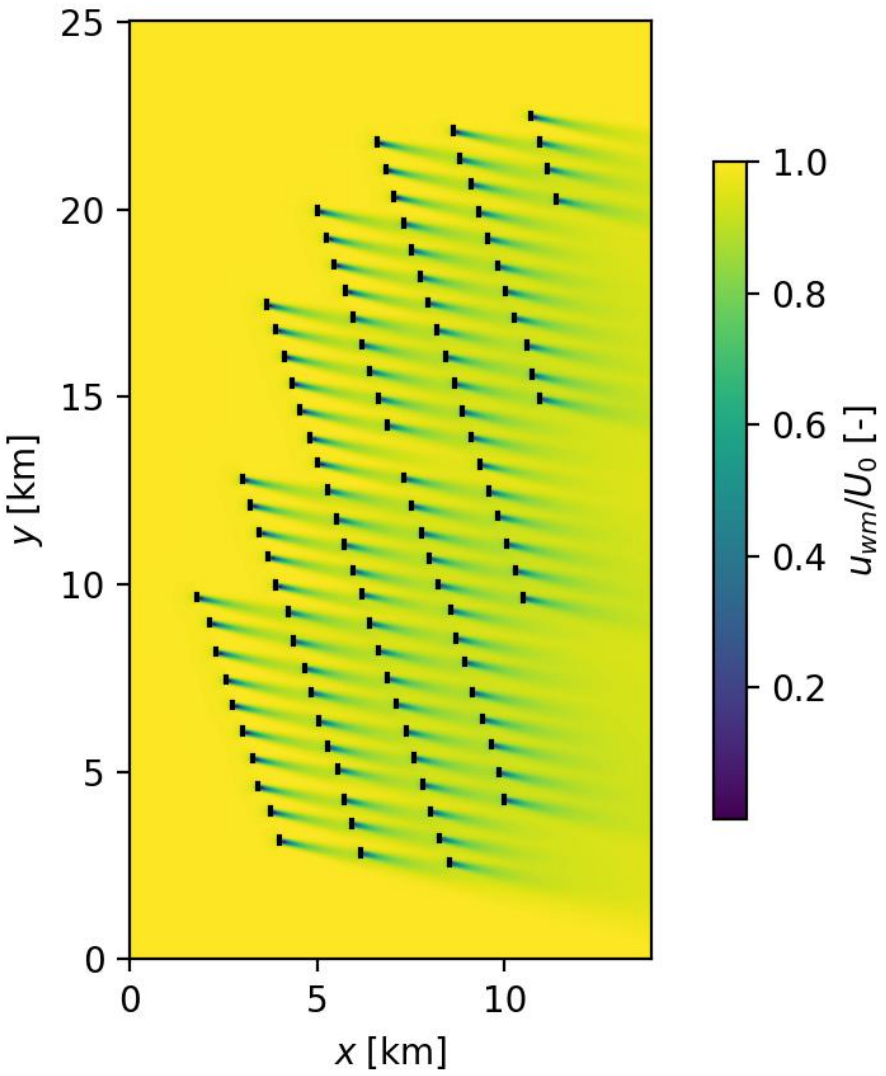
2scm:

$$\partial_t \mathrm{P}^{ij} = \frac{1}{2} N^{ij} \rho C_P^* \|V_{ij}^{REWS}\|^3 A_F$$

$$\partial_t \mathrm{TKE}_{\mathrm{drag}}^{ijk} = \frac{1}{2} \frac{N^{ij} C_{TKE}^* \|U_{ij}^{REWS}\|^3 A_{ijk}}{\Delta z_k}$$

Wake Model computation

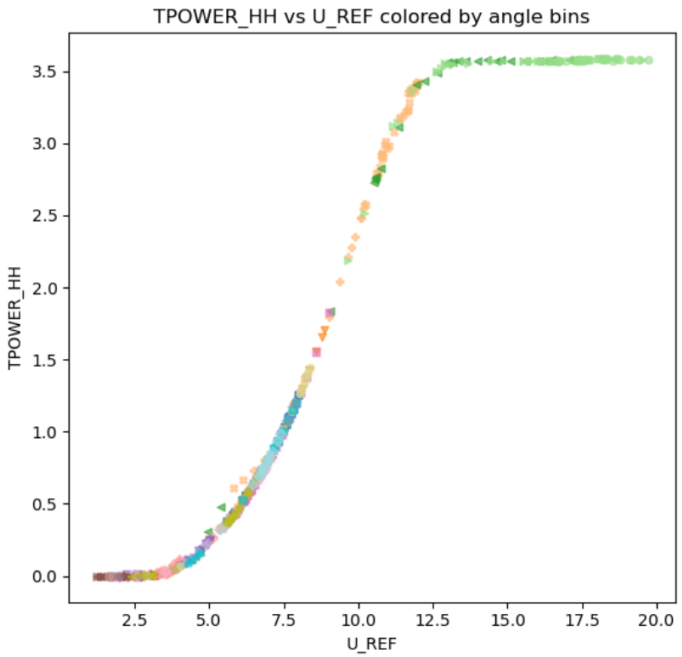
- Wake Model underlying WAYVE
- Get turbine positions inside CCLM grid
- Compute all values at one
- One LUT per cell storing REWS, CP_ST, CT_ST

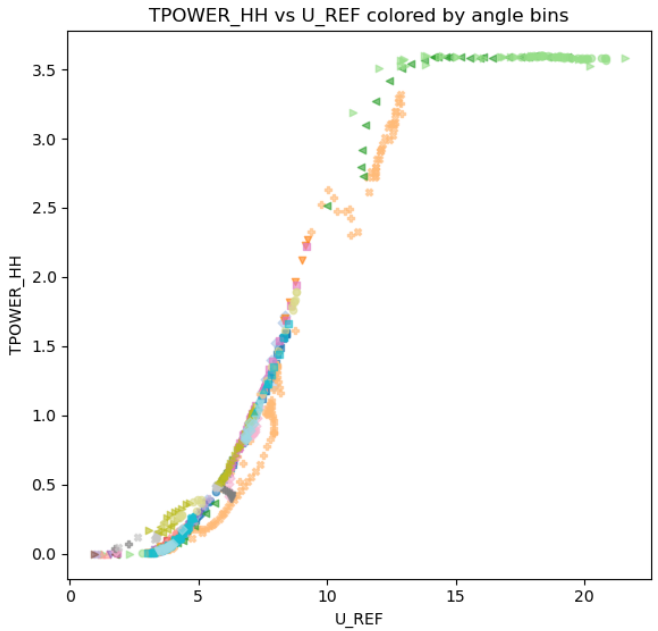


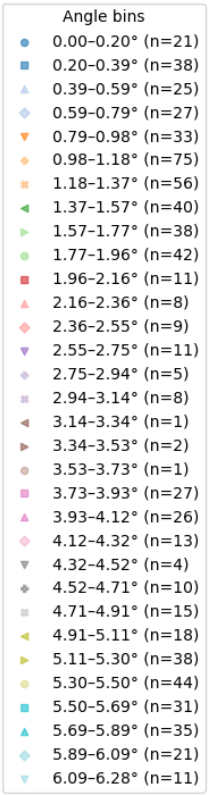
```
Cell index [i,j], i longitude and j latitude index
156 187
Number of inflow velocities, angles and TKE:
7 8 1
inflow Speeds:
0.00 5.00 10.00 15.00 20.00 25.00 30.00
inflow Angles:
0.00000000e+00 7.85398163e-01 1.57079633e+00 2.35619449e+00 3.14159265e+00 3.92699082e+00 4.71238898e+00 5.49778714e+00
inflow TKE's:
1.10000000e+00
REWS per angle:
9.96527374e-05 9.96527374e-05 9.96527374e-05 9.96527374e-05 9.96527374e-05 9.96527374e-05 9.96527374e-05 9.96527374e-05
4.89111418e+00 4.88745986e+00 4.69560871e+00 4.33646191e+00 4.38591238e+00 4.52906856e+00 4.86285102e+00 4.89692644e+00
9.77059125e+00 9.76329467e+00 9.37973110e+00 8.67396841e+00 8.76640476e+00 9.04767120e+00 9.71436960e+00 9.78260648e+00
1.47898477e+01 1.47858663e+01 1.45864940e+01 1.40736252e+01 1.42252101e+01 1.44008888e+01 1.47610283e+01 1.47945318e+01
1.97457783e+01 1.97435874e+01 1.96331752e+01 1.93620883e+01 1.94360005e+01 1.95304925e+01 1.97299918e+01 1.97484664e+01
2.46835104e+01 2.46820498e+01 2.46082021e+01 2.44297266e+01 2.44768971e+01 2.45395583e+01 2.46729856e+01 2.46853210e+01
2.96256053e+01 2.96256053e+01 2.96256053e+01 2.96256053e+01 2.96256053e+01 2.96256053e+01 2.96256053e+01 2.96256053e+01
THRUST_COEFF
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
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2.52572329e-01 2.52708641e-01 2.62745404e-01 2.95831046e-01 2.81547808e-01 2.74025770e-01 2.54116300e-01 2.52412422e-01
1.05947519e-01 1.05971058e-01 1.07490463e-01 1.12044737e-01 1.10262741e-01 1.09134144e-01 1.06173937e-01 1.05918679e-01
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POWER_COEFF
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4.48155677e-01 4.49146166e-01 4.62492569e-01 4.60999603e-01 5.02004110e-01 4.73717065e-01 4.46921126e-01 4.46506398e-01
1.90661729e-01 1.90815827e-01 1.99108950e-01 2.21291746e-01 2.14988471e-01 2.07200107e-01 1.91854196e-01 1.90480691e-01
8.02860395e-02 8.03127734e-02 8.17230261e-02 8.52979698e-02 8.43004796e-02 8.30770267e-02 8.04878068e-02 8.02532588e-02
4.08853850e-02 4.08926464e-02 4.13038022e-02 4.24049021e-02 4.20465028e-02 4.17174760e-02 4.09450336e-02 4.08763889e-02
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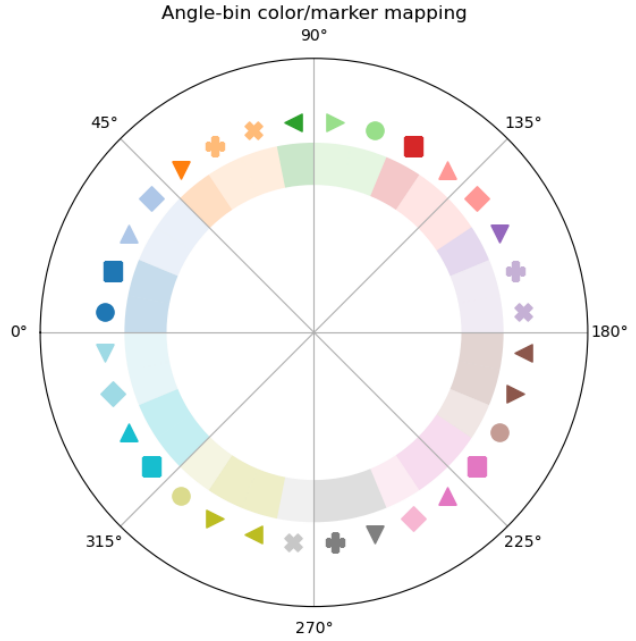
Fitch versus 2scm Power Curve

- Empirical power curve averaged over the farm



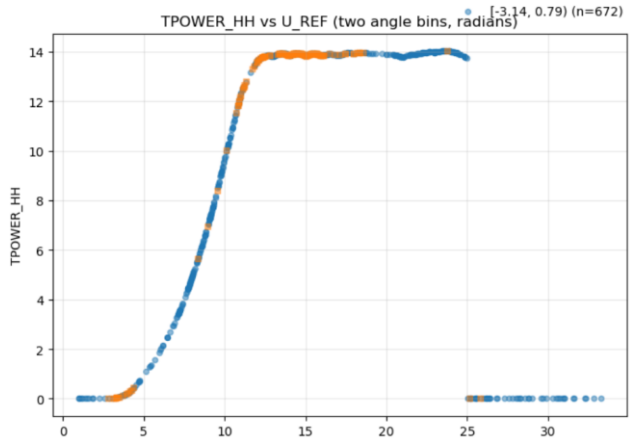


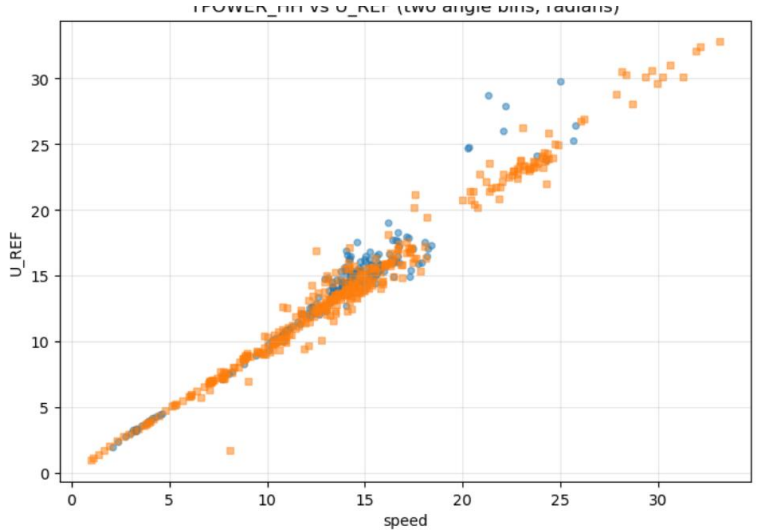


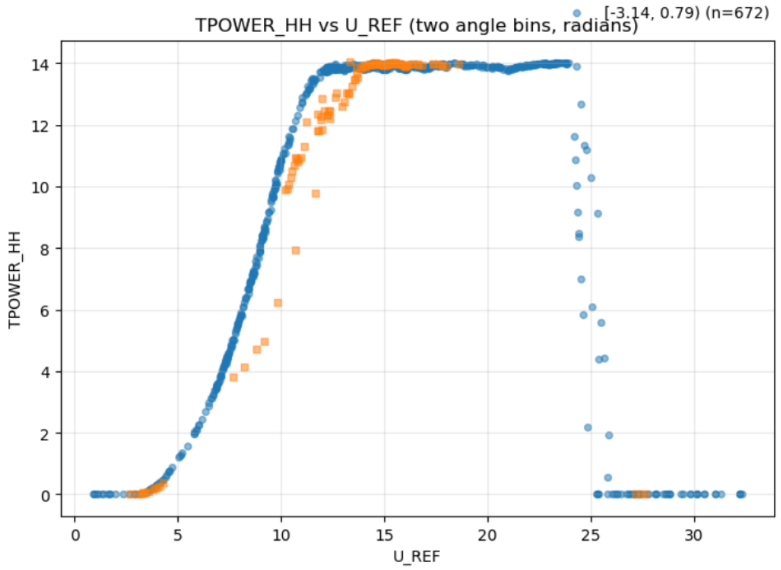


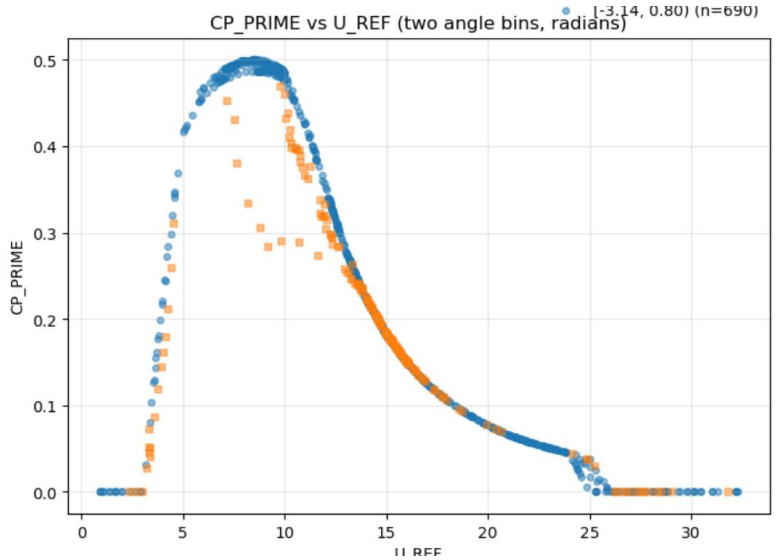
One cell farm

- Angles in between 0.8 and 1.2 are affected by lowe prime coefficient
- For these angles, reference velocities are slightly higher compared to the layer velocity than else
- Therefore, lower power production only cause of lower prime coefficient

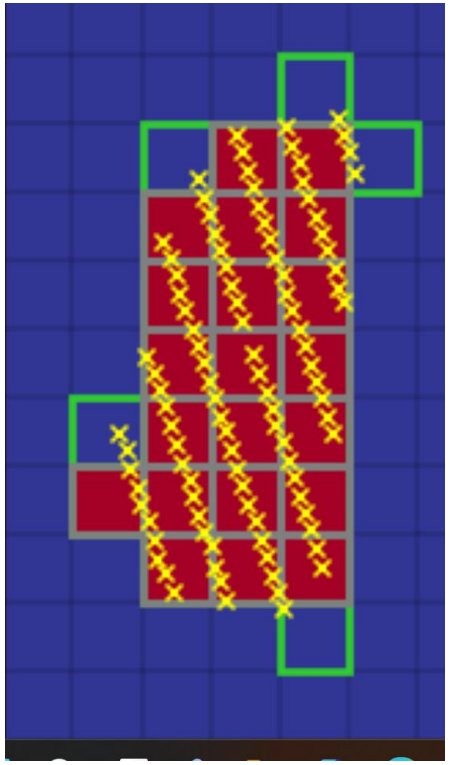


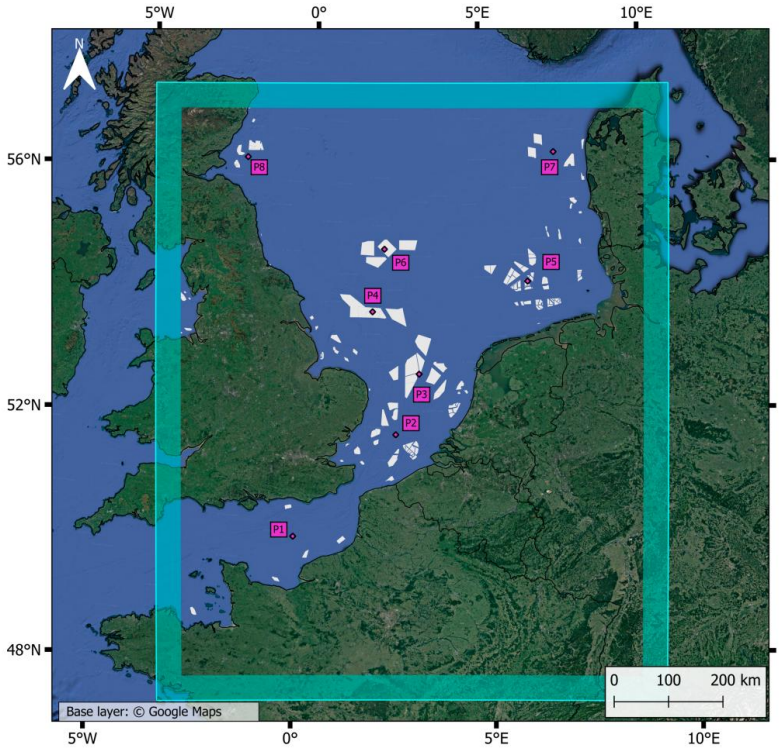






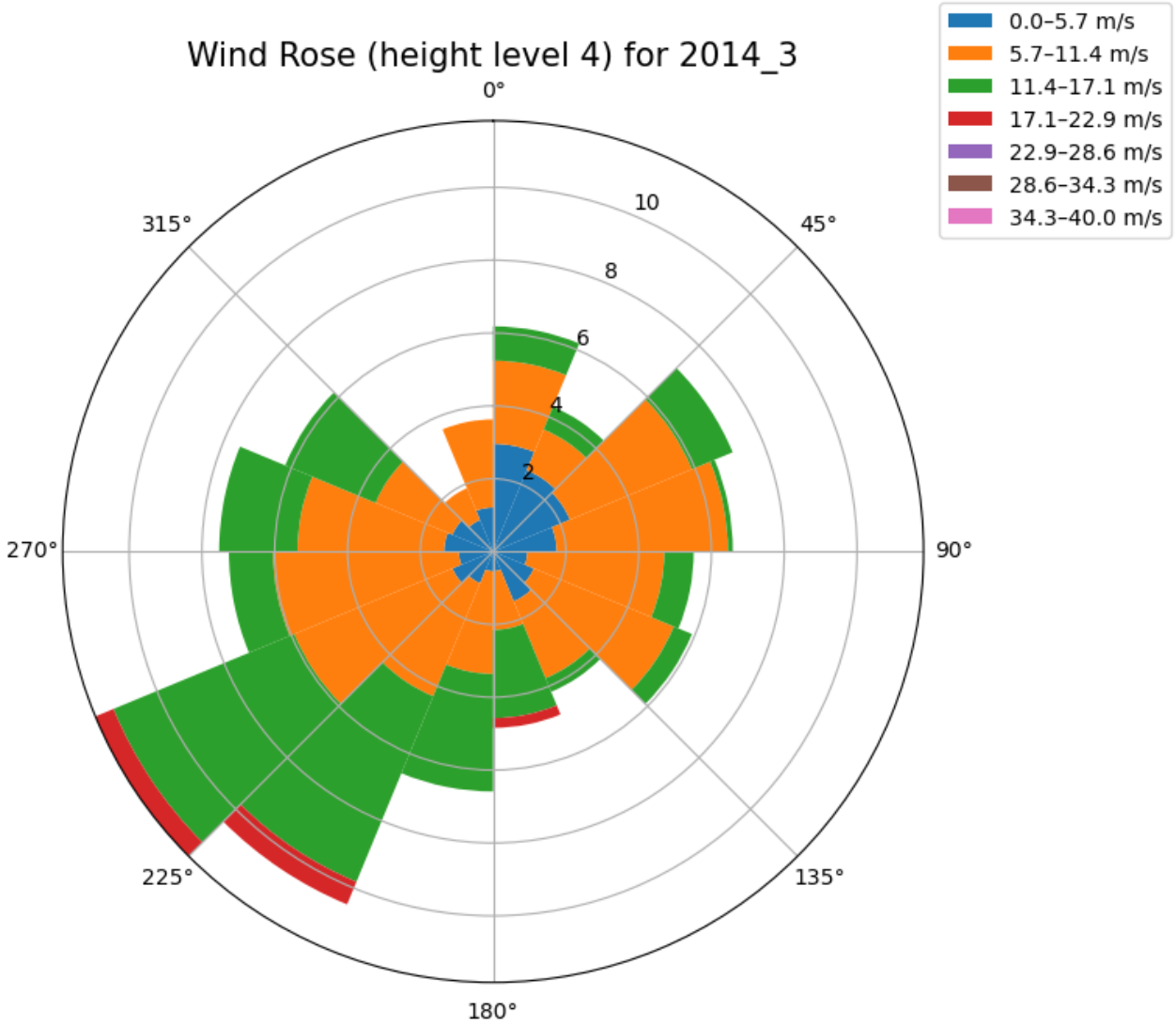
1 Month simulation

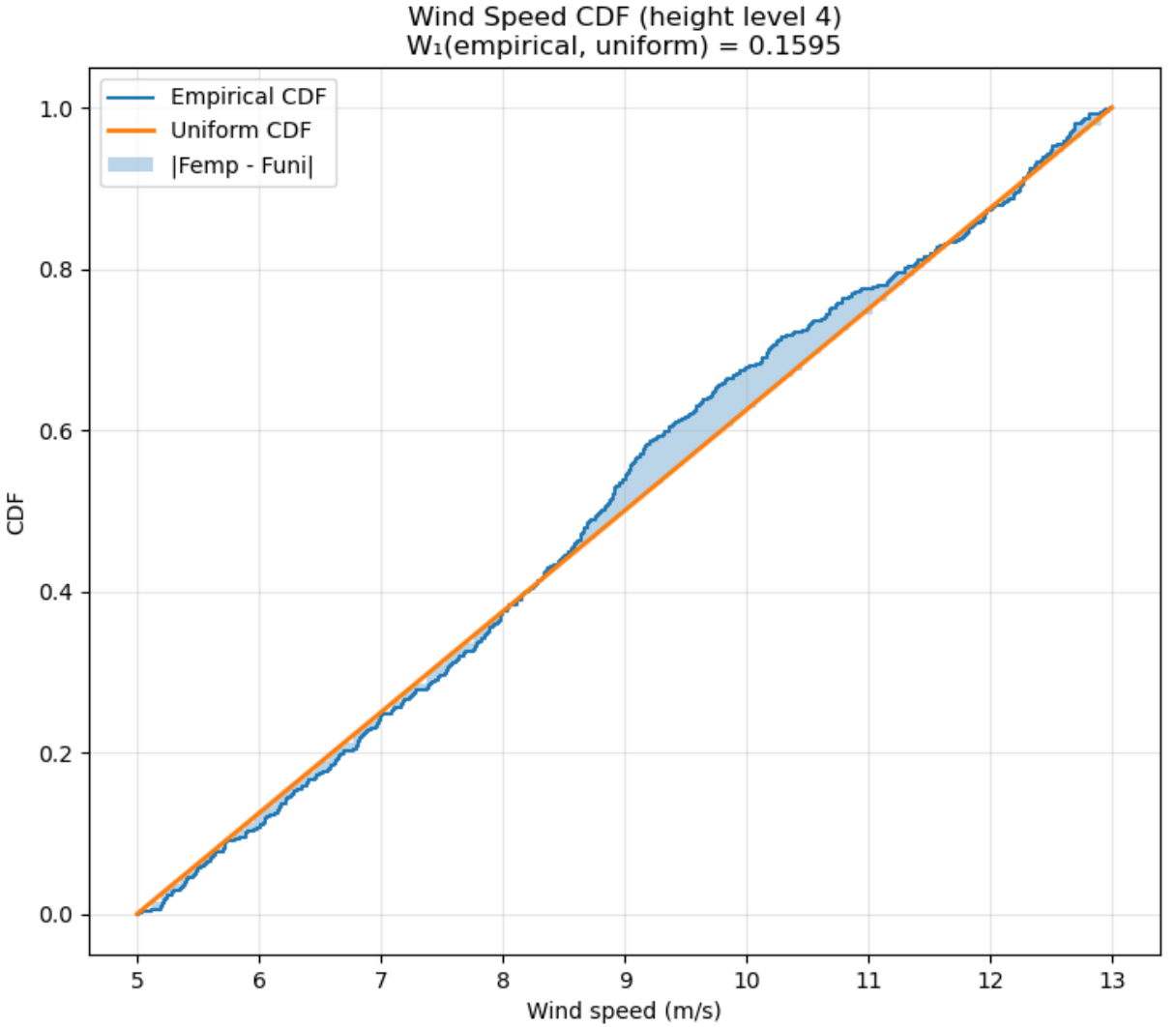




Evaluation Methods

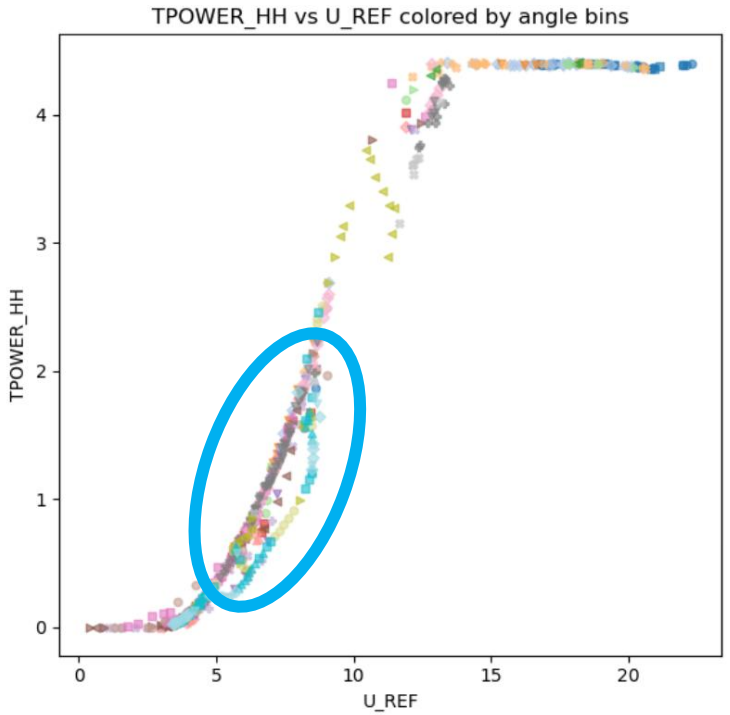
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Rank 1: 2014_03
    min_angle_bin = 7, rank = 2
    min_speed_bin = 48, rank = 2
    wst_dist      = 0.15948541462421417, rank = 2
```

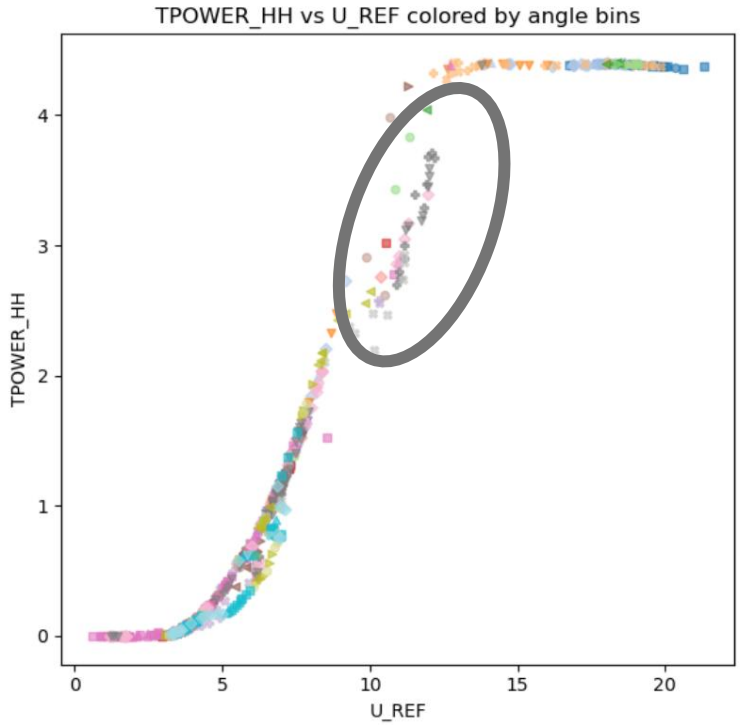


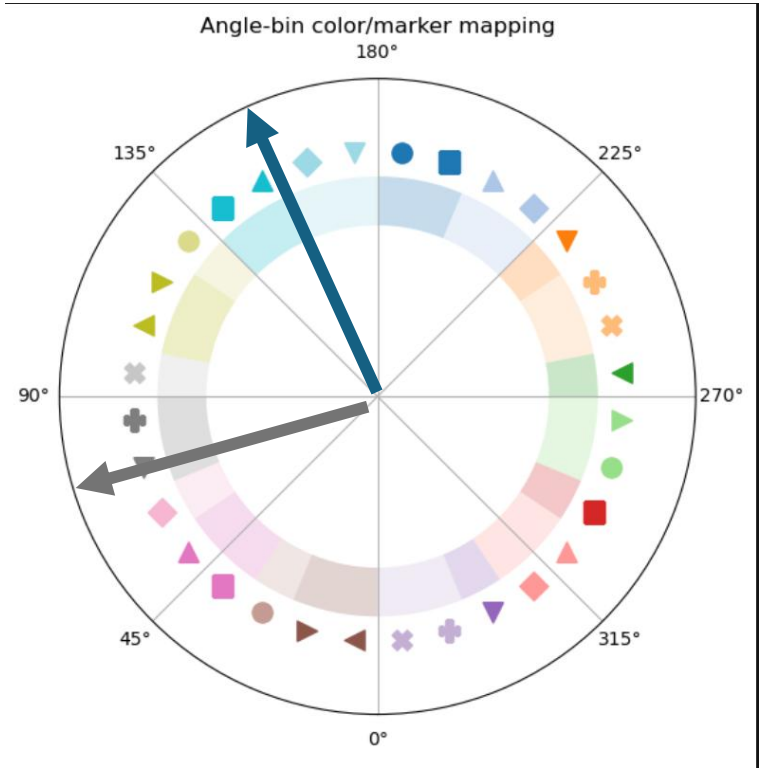


Bottom left and top right average

- NNW, NWW, WWS most waked directions

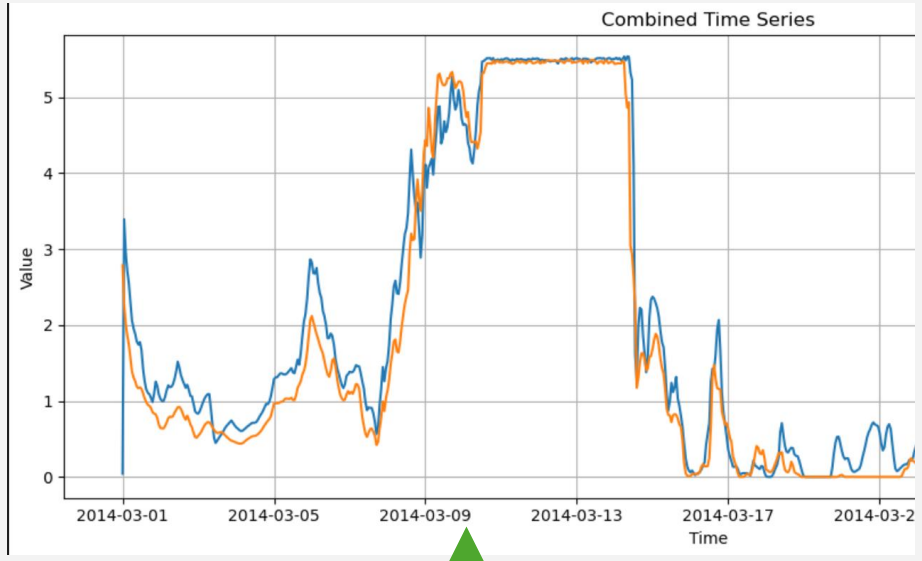


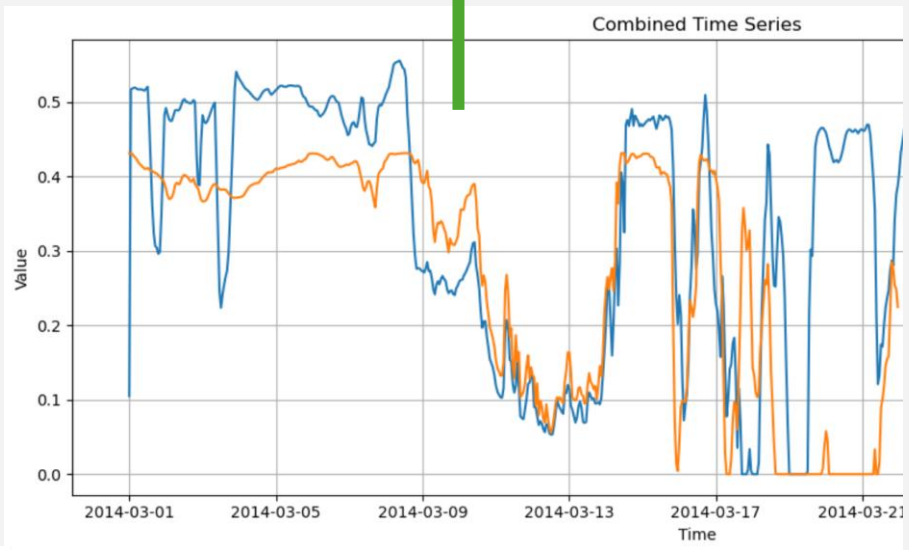


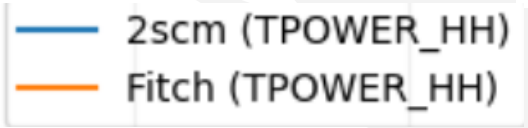


Power total and power coefficient curve over time

- Curves for cell in middle of farm
- If not wind from specific direction, cp star dominates

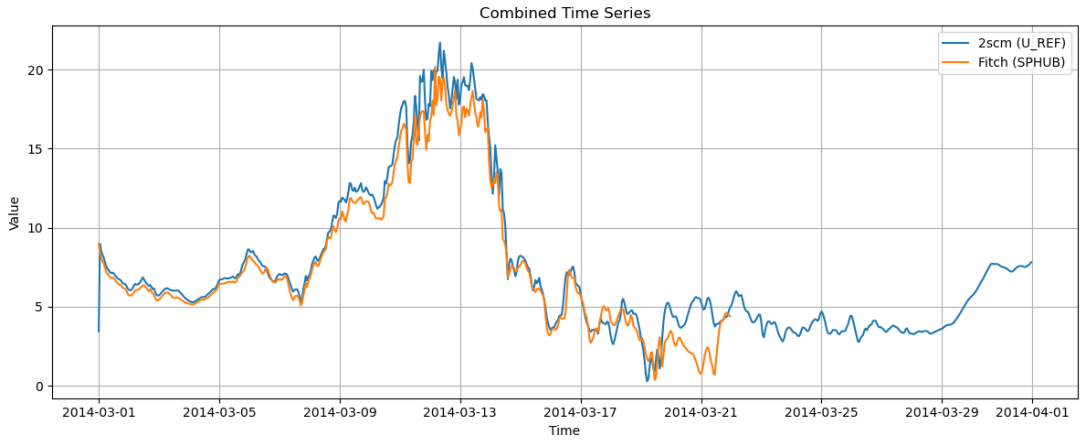


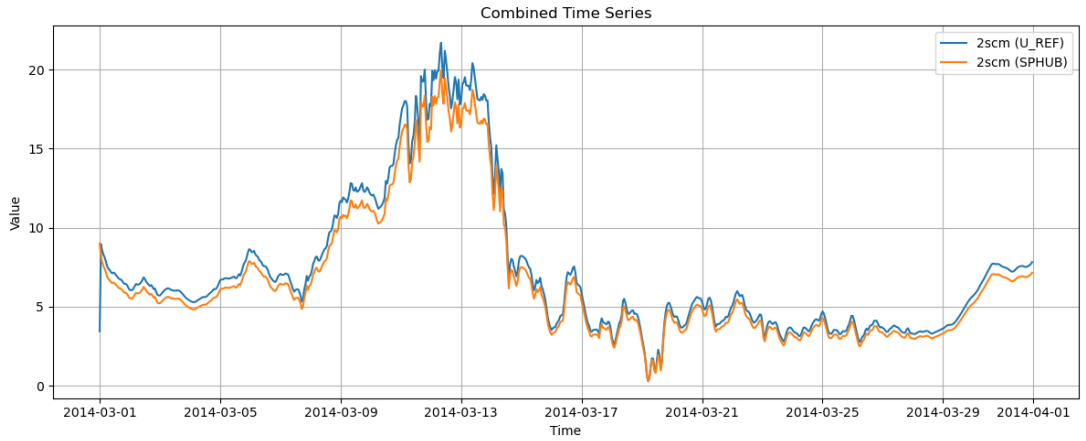




REWS and SPHUB

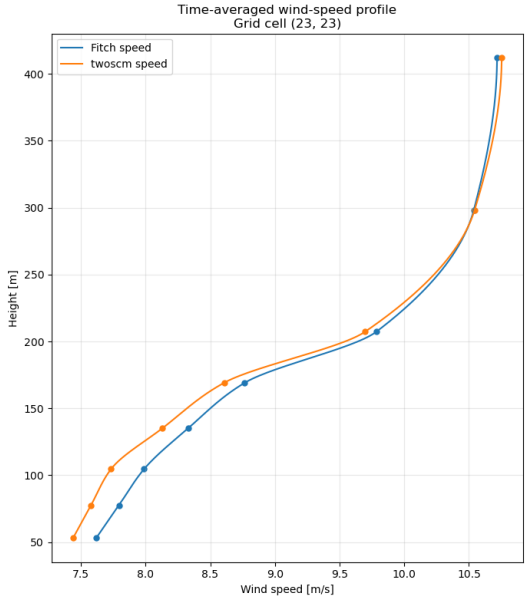
- Fitch’s sphub larger than rews in 2scm
- 2scm’s sphub smaller than rews
- Linked to higher power extraction

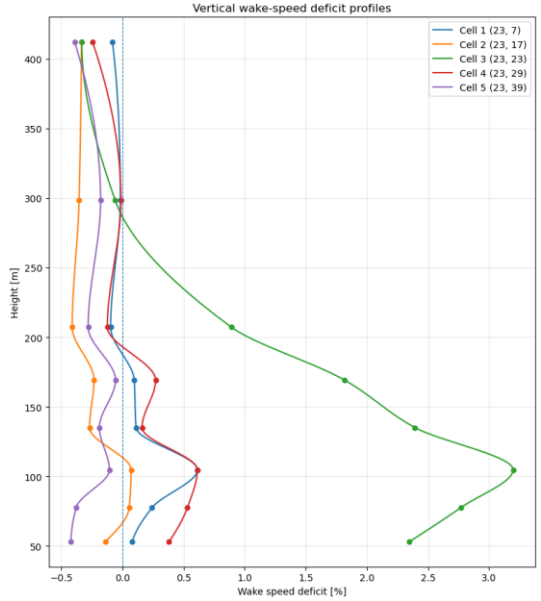


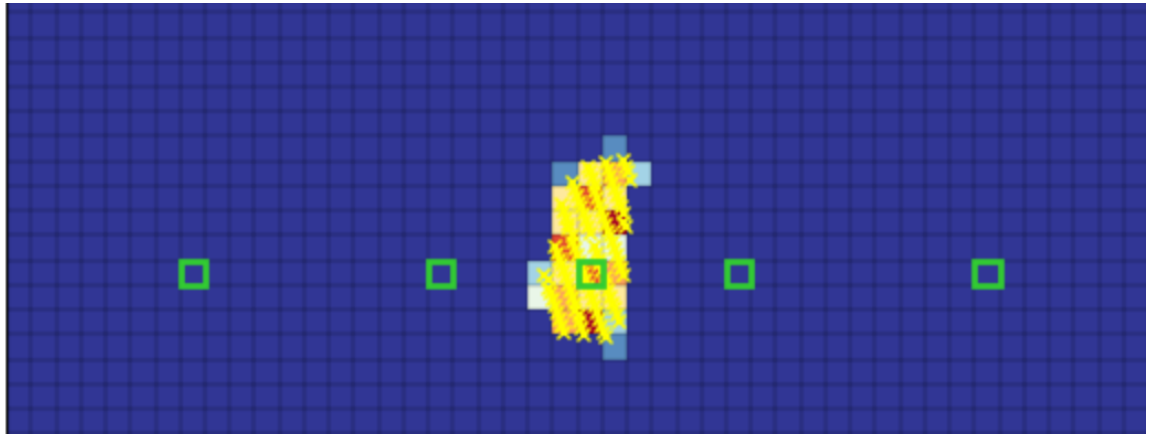


Relative Wake deficit

- 4 cells and 14 cells from farm up and downstream
- Farm center: 2scm creates slower velocities at turbine levels
- Fitch creates slightly higher wind speeds in mid levels
- Bottom plot averages height profile in farm center

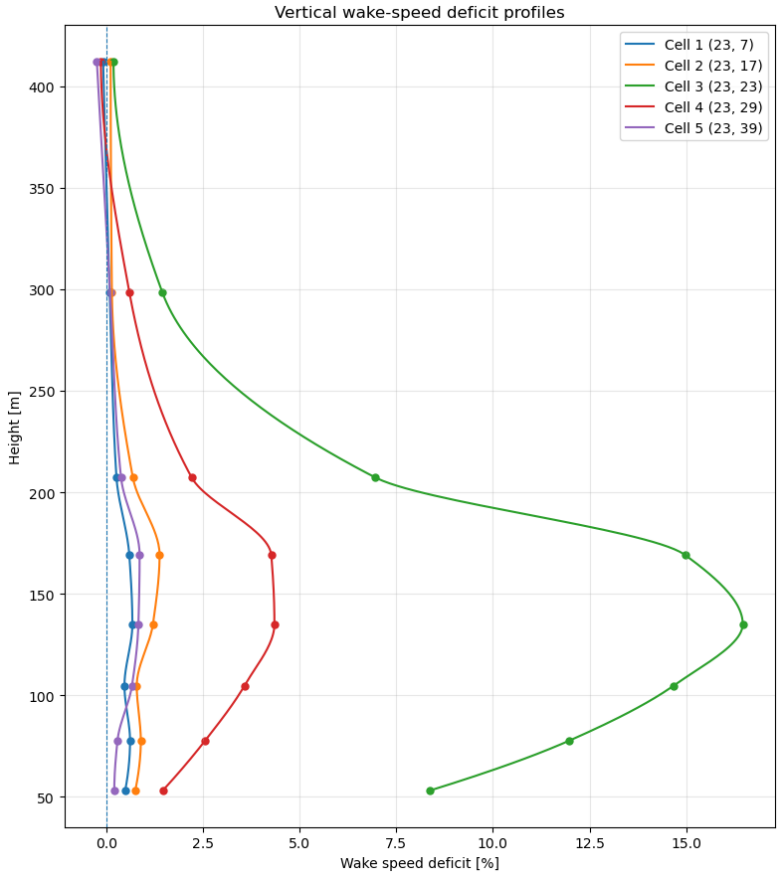


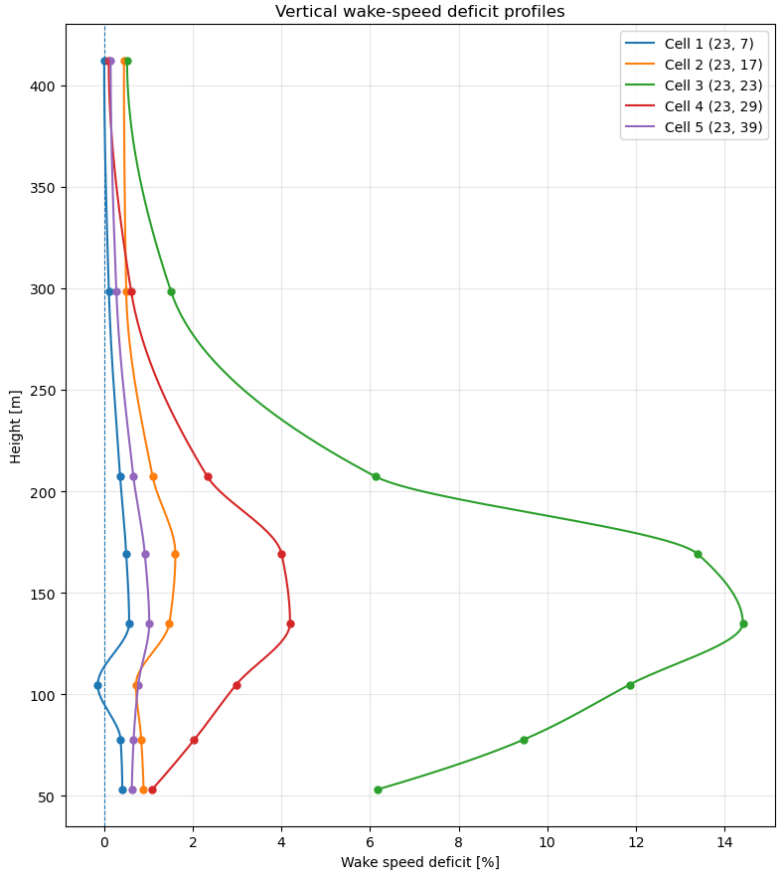




Absolute wake deficit 2scm versus fitch

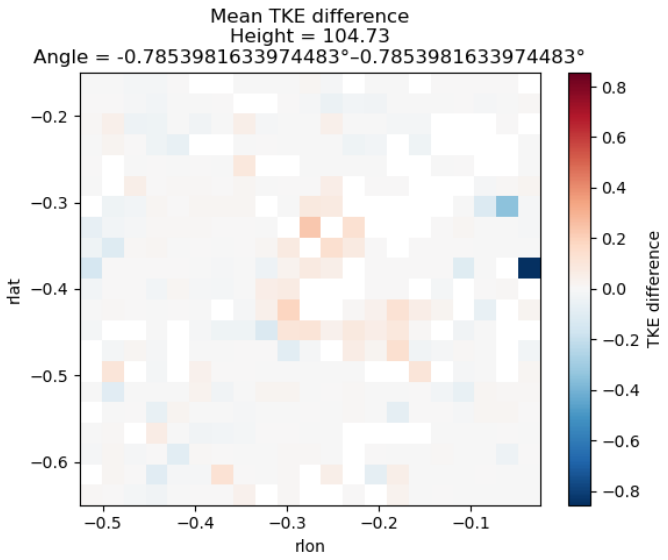
Left: 2scm. Right: Fitch

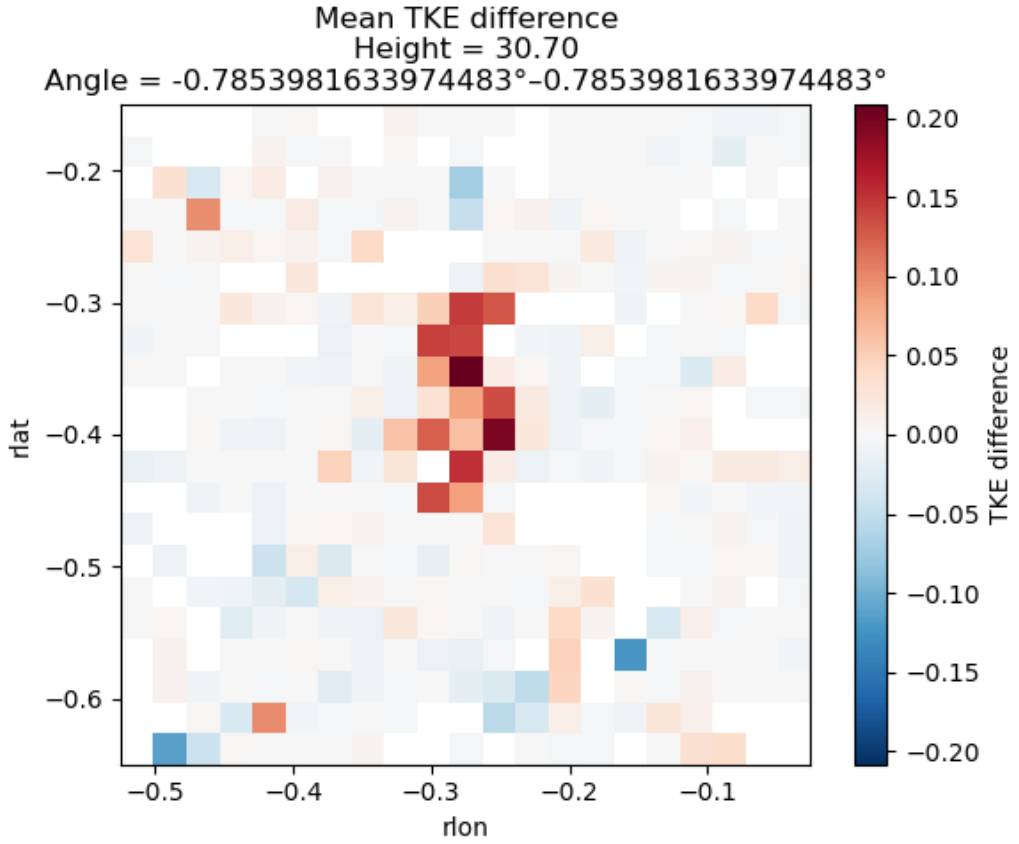




TKE enhancement for lower levels

- Left: 100 meters height
- Right: 30 meters height
- Westerly winds



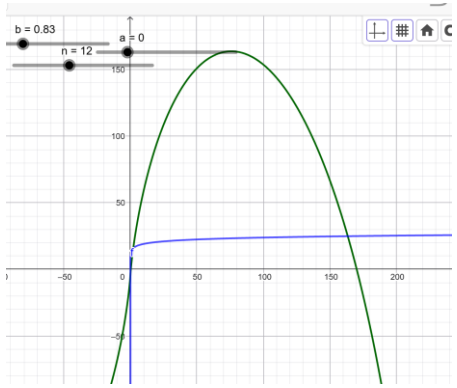


Logarithmic Wind Inflow in Wake Model

- Logarithmic Wind profile $$U(z) = \frac{u_*}{\kappa} \ln\left(\frac{z - z_0}{z_0}\right)$$

- Using Charnocks relation, $$z_0 = \frac{\alpha_c \, u_*^2}{g}$$ one can compute numerically the friction velocity for which a certain hub height wind speed hold

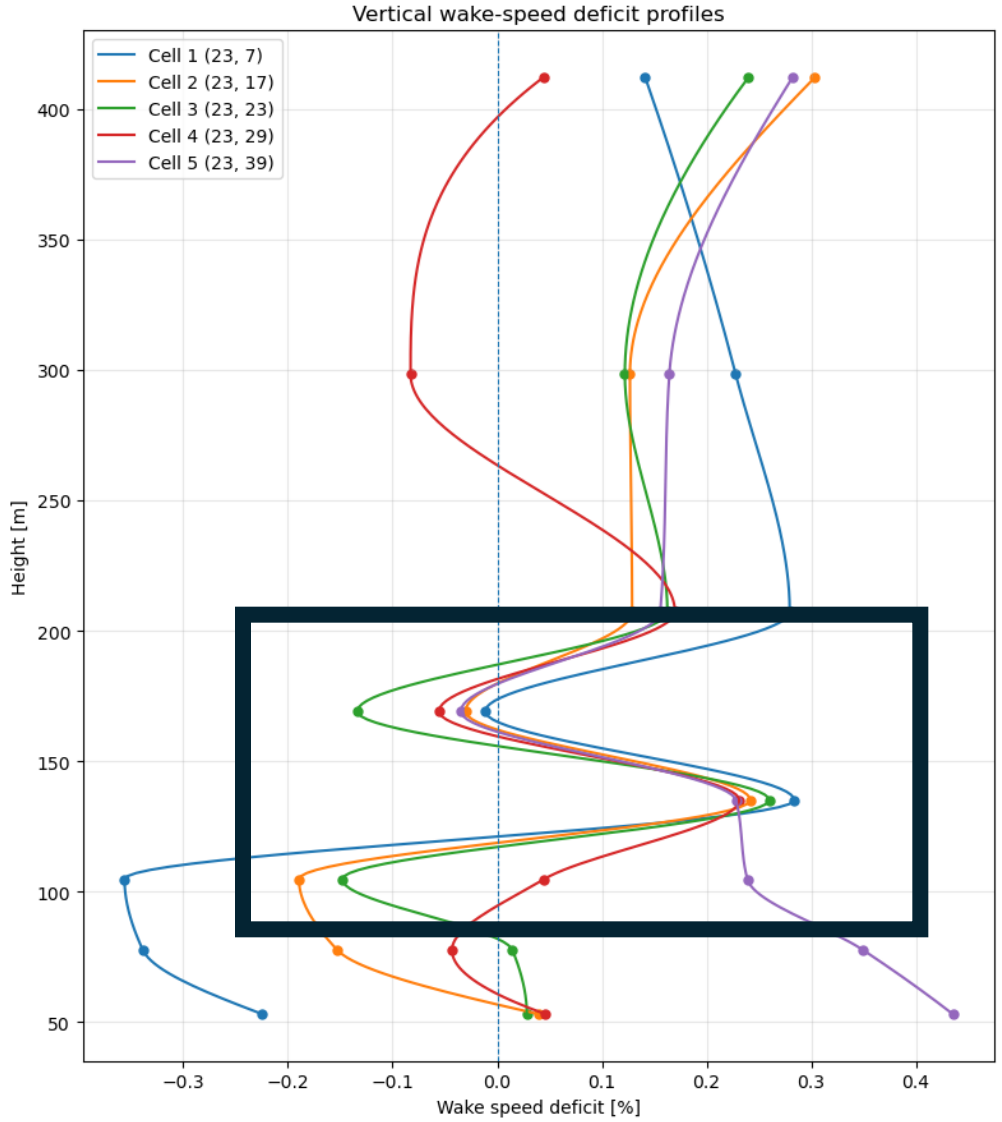
$$\text{Finding } u_* \text{ so that } U(h_T) = U_h \Leftrightarrow \text{ Solving } 0 = u_* \ln\left(\frac{g h_T}{\alpha_c} u_*^{-2} - 1\right) - \kappa U(h_T) \text{ for } u_*.$$



```
STARTING WAKE MODEL SIMULATION
Inflow Speed: 0.0 ; Friction Velocity: 0.000
Inflow Speed: 5.0 ; Friction Velocity: 0.137
Inflow Speed: 10.0 ; Friction Velocity: 0.307
Inflow Speed: 15.0 ; Friction Velocity: 0.497
Inflow Speed: 20.0 ; Friction Velocity: 0.701
```

Using logarithmic profile in Wake model

- Observations on small scale (0.1-0.5%)
- More concentration of speed towards hub height



Work package overview

Months 1 – 18: Parametrization development

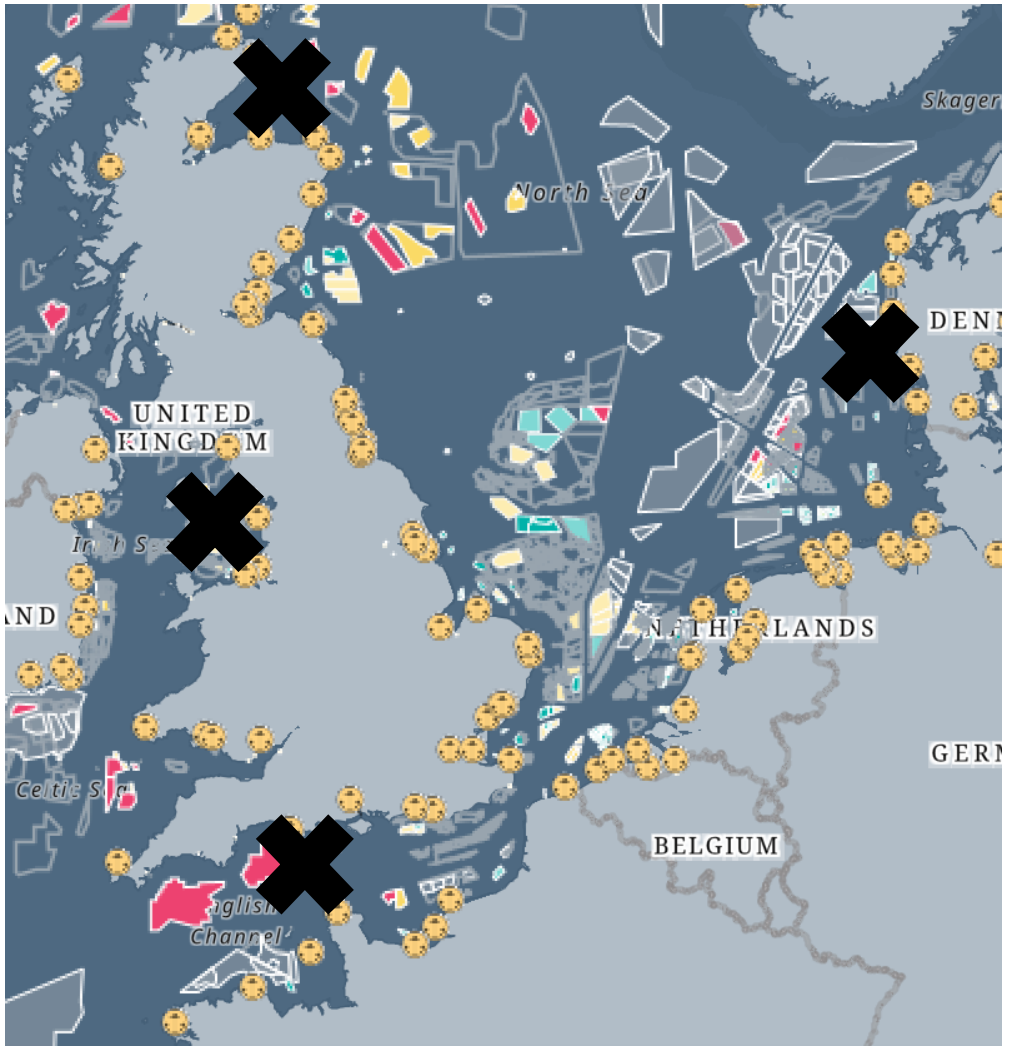
Months 12 – 25: Model validation – cooperation with industrial partners

Months 18 - 36: downscaling

Months 36 – 48: evaluation of farm design choices – cooperation with industrial partners

New Domain

- Comp Factor 25-30% increased
- Depending on spin up zone to the west
- Tier 1 cluster risk → create several EXT
- Run of 10 years ca. 45 days



Industrial Cooperation



hand in model output

They evaluate with SCADA data and compute statistics

Power per turbine row	Angle dependance of power	Velocity and TKE distribution in farm

WP3: Overview

2 Layout expansions ,

1 no farm layout

1-2 GCM forcings

1995-2025 &

2026 - 2055

Data Analysis to compare with Borgers


Thank you for your attention